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Optimal Utility of Satellite Constellation Separation with Differential Drag

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As small satellite constellations become more frequent, questions as to their deployment and orbit maintenance becomes more critical. Nanosatellites are often deployed together, do not have the luxury of on-board propulsion, and hence must reach their desired orbits by other 'passive' means, such as differential drag. The problem is compounded by other factors, such as ground station availability and access, as well as changing atmospheric densities. We present here the optimal control problem solved (with constant atmospheric densities) for varying utility functions such that satellites utilizing differential drag are phased at equal angular distances. During actual operations, a simplified scheme might be preferred. As such, a Taylor series approximation method to analyze a bang-bang control scheme is formulated and compared with the optimal solution. The final solution is not drastically affected by modest varying factors in drag and drag area, but is significantly affected by starting altitude as expected. The solution can be extended to any number of satellites, although the solution does suffer from numerical degradation the longer it takes the constellation to achieve its final configuration.

Nomenclature

$\vec{a}_D$	Acceleration (km/s^2)
C_D	Drag coefficient
A	Area (m^2)
m	Mass (kg)
ρ	Atmospheric density (kg/m^3)
v	Velocity (km/s)
v_{rel}	Relative velocity (km/s)
v_0	Initial velocity (km/s)
n	Mean motion (rad/s)
P	Orbital Period (s)
μ_E	Earth's gravitational parameter (km^3/s^2)
a	Orbital radius (km)
θ	Separation angle (rad)
θ_{target}	Target separation angle (rad)
t	Time (s)
n_{sat}	Number of satellites
$_i$	Index number for satellites
J_2	2nd spherical harmonic term
f	Utility function (rad·s)
f_{max}	Maximum utility achievable (rad·s)
L	Operational orbital lifetime (s)
τ	Non-dimensionalization of time
ω	Non-dimensionalization of velocity
Θ	Non-dimensionalization of angle
LEO	Low Earth Orbit
ISS	International Space Station
ADCS	Attitude Determination and Control Systems
AGI	Analytical Graphics, Inc.
STK	Satellite Tool Kit
HPOP	High-precision Orbit Propagator
JB2006	Jacchia-Bowman 2006 Atmospheric Model
SBB	Singular bang-bang
HBT	Hyperbolic Tangent

I. Introduction

The concept of spacecraft orbit control by means of differential drag, using attitude maneuvers to change the presented drag area, has long been known and postulated. Much work has been done in the area of formation-keeping and rendezvous [1-3], and recently of more than simply 2 spacecraft [4]. Most of these methods rely on the linearized Hill equations, assuming that the relative distance between satellites is small compared to their orbital radii. Further incorporations of the J_2 effect into these linearized dynamics are introduced by Schweighart and Sedwick and utilized in multiple spacecraft rendezvous [5].

However, with the advent of CubeSat technologies, there is a need to launch not just tight clusters of nanosatellites that maintain their relative positions, but the need to separate them out over time. CubeSats in themselves are restricted by power and size, and hence often do not have the luxury of onboard propulsion. New developments in Attitude Determination and Control Systems (ADCS) have allowed for some measure of control in attitude, and hence the ability to rotate solar panels in and out of the velocity ram direction to control drag area.

Initial studies in the area of deploying multiple Low Earth Orbit (LEO) satellites in a constant phased configuration about the Earth have been recently explored [6,7]. Furthermore Aerospace Corporation have performed on-orbit feasibility studies in this area and found CubeSats to have the necessary capability to utilize drag effects successfully [8].

The orbital maintenance of a multiple CubeSat mission can generally be divided into two portions:

1) Acquisition phase: satellites drift relative to each other to separate into their nominal orbits given time/fuel constraints.
2) Station-keeping phase: keeping the satellites at their nominal locations relative to one another given disturbances such as drag, J_2, etc.

As stated previously, many studies have explored the station-keeping phase of missions quite thoroughly. However, an optimal control formulation addressing the acquisition phase of co-deployed satellites along their orbit has not been widespread, but has previously been posed [9].

This paper takes the acquisition problem and formulates it in an optimal control setting. The utility function can be seen as a combination of the time it takes for all satellites to achieve their nominal orbits and overall lifetime they must sacrifice to attain their final configurations. The utility function also should attempt to maximize the remaining operational lifetime of a satellite orbit given the above conditions.

An analytic Taylor series formulation of the satellites' motion is employed as a tool for exploratory analysis to identify the theoretically optimal control solution in a constant (across time and altitude) atmospheric density setting under a circular orbit assumption. This approach is shown to approximate the actual dynamics of the satellites quite closely, and was selected for the following motivations:

1) Allows inclusion of atmospheric density variability over time (i.e. from atmospheric models and historic space weather observations), although density is not updated when comparing control approaches in this paper.
2) Allows investigation of simple control methods such as 'singular bang-bang' (henceforth referred to as SBB) where each satellite is constrained to only one 'maximum drag' maneuver in attempting to reach its target orbit phasing.
3) Fast orbital propagation, which enables validation of general fleet activities over long durations.

Propagation accuracy degradations are expected, along with inaccuracies in atmospheric density predictions, but this approach allows quick and meaningful comparisons of control strategies and gives a general idea of constellation maintenance operations.

The paper is organized as follows: Section II describes the orbital mechanics involved along with the Taylor series formulation. Section III poses the optimal control problem as well as the SBB problem, and solves each for 3 alternative utility functions. Section IV reports the results of numerical simulations to assess the inaccuracies of the Taylor approach. Finally, conclusions and future work prospects are outlined in Section V.

This work was primarily motivated by Planet-Labs' Flock 1 mission to launch a constellation of Earth observation satellites in 2014. Initial deployment is planned from the International Space Station (ISS), while future launches are planned to higher altitudes [10]. The satellites closely resemble the 3U form factor, and will be 3 axis stabilized with two deployable solar arrays. A simple attitude maneuver allows for a 6:1 area ratio between the maximum and nominal drag configurations, allowing the effective use of differential drag in low orbits. On-orbit data will be collected for verification of this model as well as future analysis. Optimal control solutions were obtained using Bocop, an optimal control solver. Simulations were performed using AGI's Satellite ToolKit (STK) and Orekit, which is a low level space dynamics library written in Java.

II. Dynamic Model

The use of differential drag to separate satellites is not a new concept, as it is known that satellites with differing areas will undergo different accelerations as described by

$$\vec{a}_D = \frac{dv_{rel}}{dt} = -\frac{1}{2} C_D \frac{A}{m} \rho v_{rel}^2 \frac{\vec{v}_{rel}}{|\vec{v}_{rel}|} \qquad [1]$$

where $\vec{a}_D$ is the acceleration due to drag, C_D the drag coefficient, A the exposed cross sectional drag area, M the mass, ρ the atmospheric density, and v_{rel} the relative velocity of the satellite to the atmosphere.

Due to drag providing a negative acceleration to the satellite's direction of motion, the initial response is for the satellite to decelerate, but due to the loss of energy and decay in the orbit, the net effect is for the satellite to gain velocity (also commonly known as the satellite paradox). The satellite's acceleration in the direction of its motion has been found to be the same as if the air drag force, reversed, were pushing the satellite [11]. This simply alters equation 1 such that there is no negative sign.

The mean motion of a satellite in a circular orbit can be expressed as

$$n = \frac{2\pi}{P} = \sqrt{\mu_E} a^{-\frac{3}{2}} = \frac{v^3}{\mu_E} \qquad [2]$$

where n is the mean motion, P the orbital period, μ_E Earth's gravitational parameter, a the orbital radius, and v the orbital velocity. As can be seen, satellites at higher orbits will possess lower velocities and thus lower mean motion rates. Hence satellites at lower altitudes will drift faster along-track than satellites at higher altitudes. We can then simply express the rate of angle change between two satellites A and B as

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{1}{\mu_E}(v_A^3 - v_B^3) \qquad [3]$$

where θ is the separation angle between satellites A and B, and v_A and v_B their respective velocities.

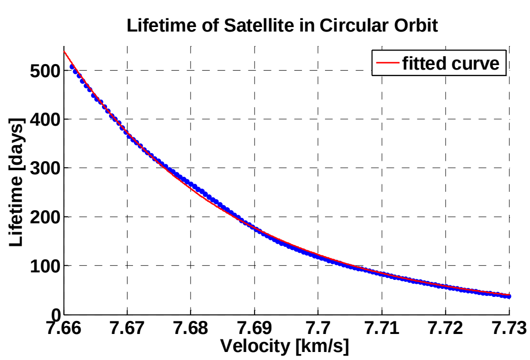


Figure 1 Lifetime of a satellite in circular orbit as a function of its velocity. Data is modeled from AGI's STK lifetime tool, fitted to an exponential curve.

The orbit phasing of two satellites using differential drag starts with both in identical initial orbits and in nominal drag configuration. Satellite A transfers to a high drag configuration and decelerates, hence dropping in altitude. Since A's altitude is now lower than that of B, the differing mean motions allows A to drift ahead of B. After some drag duration A then returns to its nominal drag configuration. At some point, B will perform a similar series of maneuvers, forcing it to drop in altitude in such a manner that it achieves the same final altitude (and therefore mean motion) as A, but now separated along-track by the specified separation angle.

The general tradeoff in such a scheme is that forcing a satellite into a high drag configuration ultimately reduces its total orbital lifetime. As orbital lifetime is innately tied to altitude, we can then treat altitude as a resource that must be conserved. In the case of a circular orbit this translate to a function of velocity, shown in Figure 1. However, we must at least perform some drag maneuver or we will never achieve our desired configuration. The objective is then to achieve the final configuration while optimizing over some utility function that depicts the total "usefulness" or value of the satellite (more details in section III).

The timing and strength (how much of the available area to present into the velocity ram vector) of these maneuvers is dictated by a number of factors, but most importantly by the atmospheric density. While a control algorithm can certainly compensate for these errors, the accuracy of future density predictions quickly degrades the further ahead of time we attempt to predict. In addition, we may need to explore a simpler class of solutions than those suggested by the optimal solution. These motivations prompt us to develop the Taylor series formulation of this problem.

If we integrate equation 1 with respect to v, we get

$$v(t) = \frac{2mv_0}{2m - C_D A \rho v_0 t} \qquad [4]$$

where v_0 is the initial velocity. However, equation 4 is only true if we hold ρ and A constant. The Taylor approximation of equation 4 yields

$$v_{i+1} = v_i \left(1 + \frac{C_D A_i \rho_i v_i}{2m} \Delta t\right) \qquad [5]$$

where we specify the velocity at the next time step as a function of values from the previous time step. Δt is the time step that we evaluate over where all values such as area and density are held constant. Similarly, the approximation for separation angle and its rate of change yields

$$\theta_{i+1} = \frac{3C_D}{4\mu m}\left(A_{1i}\rho_{1i}v_{1i}^4 - A_{2i}\rho_{2i}v_{2i}^4\right)\Delta t^2 + \left(\frac{d\theta}{dt}\right)_i \Delta t + \theta_i \quad [6]$$

$$\left(\frac{d\theta}{dt}\right)_{i+1} = \frac{3C_D}{2\mu m}\left(A_{1i}\rho_{1i}v_{1i}^4 - A_{2i}\rho_{2i}v_{2i}^4\right)\Delta t + \left(\frac{d\theta}{dt}\right)_i \quad [7]$$

The Taylor series approximation is well suited to this problem since results show that the change in separation angle rate is fairly linear, owing much to the small incremental changes in velocity experienced by such a small force such as drag. This translates to separation angle that is predominantly parabolic in profile. In order to illustrate the relative accuracy of different approaches, Figure 2 shows the difference between a full Orekit simulation with the JB2006 atmospheric model [12], the analytical solution specified in equation 4, and the Taylor series solution with various amounts of atmospheric density (constant, updated per time step, and updated with exponential atmosphere model fit) in a bang-bang control scheme. The time step chosen is one ISS orbit; roughly 92.5 minutes. Even at such a large Δt, we see the strong resemblance of the Taylor series approximation (with density updating) to the full simulation.

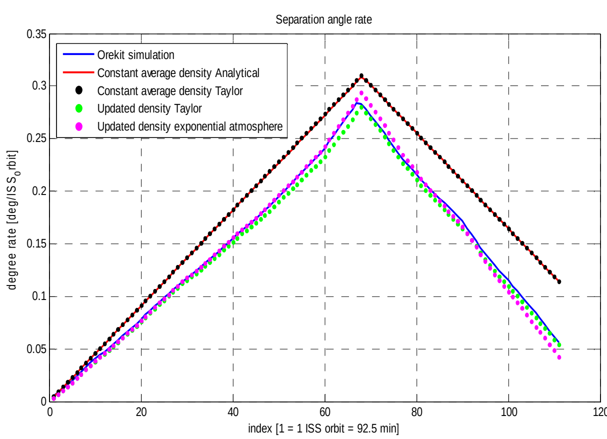


Figure 2 Comparisons of the change of separation angle rate between full Orekit simulation with full drag, analytical solution, and various Taylor series

Satellites orbiting at the same altitude in the same plane pass through the same regions of space, and are acted upon by very similar forces. The rate of change of the J_2 and solar pressure perturbations is slow relative to the mean motion, so we neglect their effect on the overall control scheme. Drag forces are minute and act on long time scales, so we anticipate that good reactionary control can be developed despite uncertainties in density, C_D, and exposed area. Figure 2 confirms that the Taylor approach is valid, however this paper predominantly focuses on comparing control schemes and utility functions, and therefore the constant average density assumption is held from now onwards.

III. Control Formulation

We can phrase the orbit acquisition problem described thus far as an optimal control problem, such that we have state equations

$$\dot{v}_i = \frac{1}{2}\frac{C_D}{m}\rho A_i v_i^2$$

$$\dot{\theta}_i = \frac{1}{\mu_E}\left(v_i^3 - v_{i+1}^3\right) \quad [8]$$

and that we maximize the total lifetime performance

$$\sum_{i=1}^{n_{sat}-1}\left[\int_0^{t_f} f(\theta_i)dt\right] + (n_{sat}-1)L(t_f)f_{max} \quad [9]$$

under the drag area constraints

$$A_{min} \le A_i \le A_{max} \quad [10]$$

where $f(\theta_i)$ is the utility function that describes the usefulness of the specified degree of separation between the i^{th} and $(i+1)^{th}$ satellite, t_f is the time it takes to reach the desired configuration, n_{sat} is the total number of satellites, and $L(t_f)$ is the useful operational orbit lifetime left after the desired configuration is attained.

The boundary conditions of this problem are such that

$$(v_i)_{initial} = v_{initial}$$
$$(v_i)_{final} = (v_{i+1})_{final} = \cdots = \left(v_{n_{sat}}\right)_{final}$$
$$(\theta_i)_{initial} = 0$$
$$(\theta_i)_{final} = \theta_{target} \quad [11]$$

Equation 11 states that all satellites start at the same velocity $V_{initial}$, that is predefined by their co-deployment orbit, and end at the same velocity which is necessary for their altitudes to be equivalent and their mean motions to be synchronized. The separation angle hence goes from 0 to the target value θ_{target}. Note that these conditions imply that the initial and final $\dot{\theta}_i = 0$.

The total lifetime performance is divided into the acquisition phase, expressed through the summation of $f(\theta_i)$, and the station-keeping phase, which assumes that the satellites can be maintained at their nominal positions relative to one another without much effort. We can alter equation 9 such that we define lifetime as an exponential function of final velocity, as shown in Figure 1:

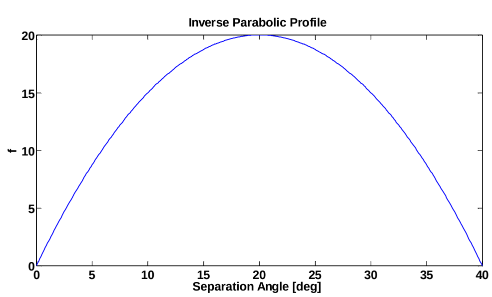
$$\sum_{i=1}^{n_{sat}-1}\left[\int_0^{t_f} f(\theta_i)dt\right] + (n_{sat}-1)c_1 exp\left(c_2 v_{1_{final}}\right) f_{max} \qquad [12]$$

This choice allows the total utility to be purely stated in terms of the state variables and the problem becomes tractable. Note that due to the extreme values in densities and the gravitational parameter careful non-dimensionalization of the problem is required, which is covered in the Appendix.

The choice of utility function f, which describes the utility of the separation angle, can take numerous forms depending on how we value θ and $\dot{\theta}_i$. Optimally, it should have properties such that

$$f(0) = 0$$
$$f\left(\frac{2\pi}{n_{sat}}\right) = \theta_{target} \qquad [13]$$

although the first condition need not be rigorously satisfied. The utility function should also ideally be continuous, but again that constraint can be dropped if necessary. We will consider a few examples: an inverted parabolic profile, a linear profile, and a hyberbolic tangent profile. Solutions are based upon a constant atmospheric density and 5 satellites using the Bocop optimal control solver with $\boldsymbol{\theta}_{target} = 20°$.



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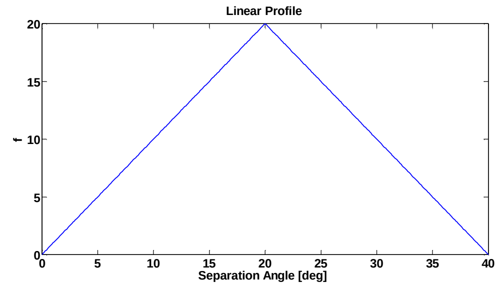

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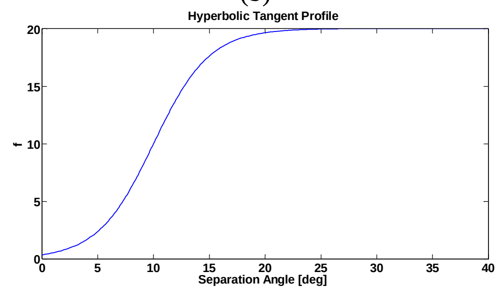


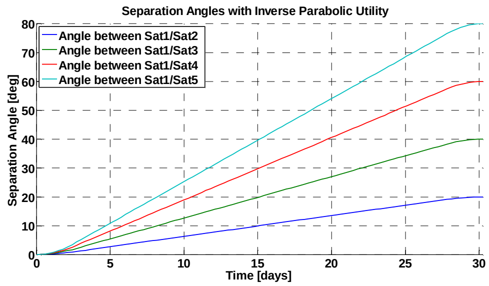
(c)

Figure 3 Various objective functions for *f(θ)* (a) Inverse Parabolic Profile (b) Linear Profile (c) HBT Profile

Figure 3 compares the three alternative utility functions considered in maximizing the total utility function of Equation 9. Each one has a slightly different connotation: the inverted parabola (a) describes a function of diminishing returns as the target value is approached, hence prolonging the drift period. The linear profile (b) is a straightforward approach where we value separation angle change evenly. Both penalize separations greater than the targeted value. The Hyperbolic Tangent (henceforth referred to as HBT) (c) is an alternative representation which does not penalize overshooting and is closer to operational reality: small separations have very little value, where as when we approach our desired separation angle, we again see a diminishing return profile.

Optimal Control

The optimal control problem sets no constraints on satellite maneuvers, and it is solved here for the three alternative utility functions by maximizing total utility over the acquisition phase. The resulting separation angles for a 5 satellite simulation are plotted against time in Figures 4, and the drag profiles (drag area vs. time) are given in Figure 5.



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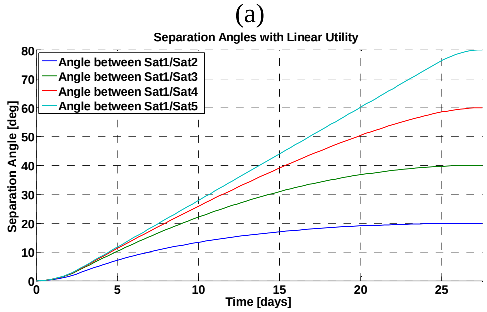

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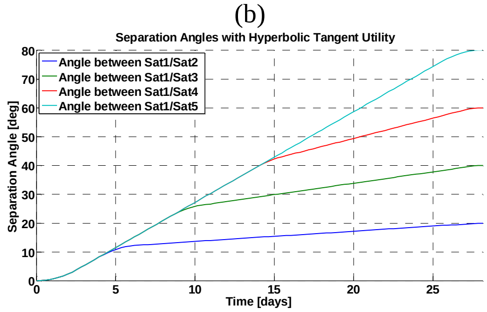
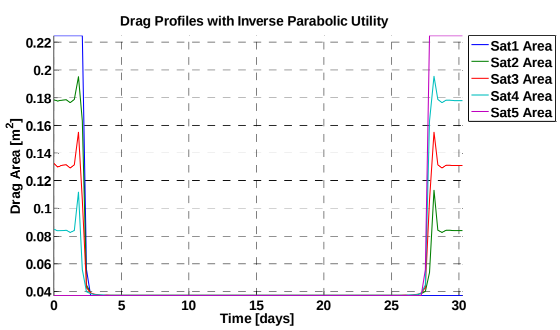


(c)

Figure 4 Separation Angle vs Time for various utility functions (a) Inverse Parabolic (b) Linear (c) HBT



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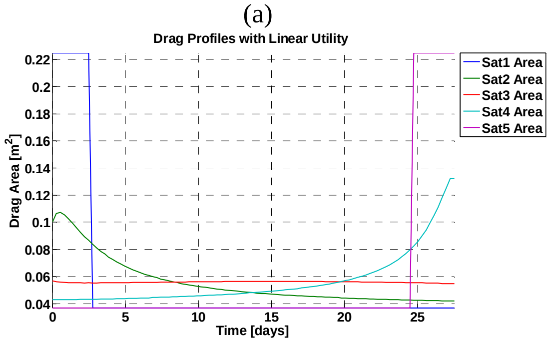

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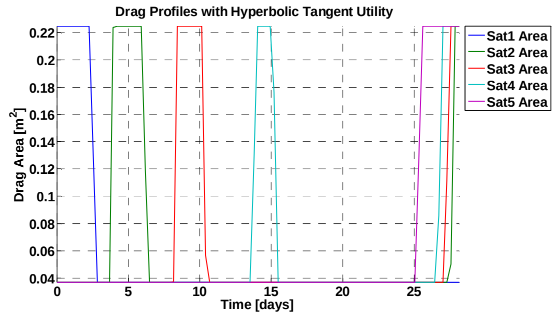


(c)

Figure 5 Drag Profiles vs Time for various utility functions (a) Inverse Parabolic (b) Linear (c) HBT. *Note: the next time step on the above plots would have all satellites at nominal drag area*

From Figures 4 and 5 we see the difference in solutions depending on our chosen utility function. Figure 4 also demonstrates that, for the three utility functions, the total time to target phasing is within about 10% of each other. The HBT solution has satellites essentially "splitting off" from the pack one at a time with drag maneuvers, and therefore lends itself to a bang-bang type of control. This is operationally far simpler than the others, where the optimal drag area falls between the nominal and max and requires more sophisticated attitude maneuvers and finer control.

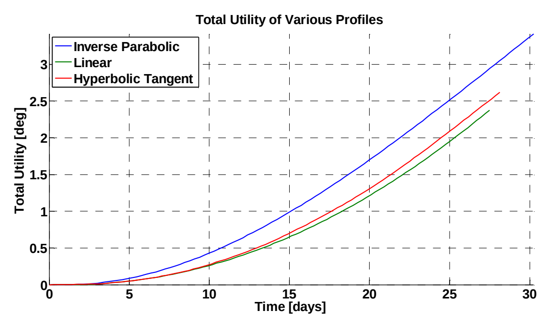


Figure 6 Total utility (Eq. 9) as seen by various profiles.

Figure 6 shows the total utility (as per Equation 9) of the various profiles. It would appear that the inverse parabolic profile results in the most total utility, but in actuality these values cannot be compared to one another as it is based upon an arbitrary transformation of the separation angle. However what we can compare is the time it takes for the methods to reach the targeted orbit phasing. As expected the inverse parabolic takes the longest, since coasting rapidly adds value at small separations. The linear and HBT solutions are closer together, with the HBT taking slightly longer due to lingering effects of a preference to coast. We can imagine if we had taken a parabolic profile, we would penalize the coasting phase and hence the optimal solution would push towards a solution that is closer to the minimum time solution. These effects are of course magnified with the addition of many satellites; the more satellites within the constellation, the larger the difference between the total utility functions.

Singular Bang-Bang Control

To minimize operational complexity, it is in our interest to develop an even simpler version of bang-bang control. We impose the following constraint: limit each satellite to only perform one maximum drag maneuver to achieve its nominal orbit. The idea is presented with 3 satellites in Figure 7.

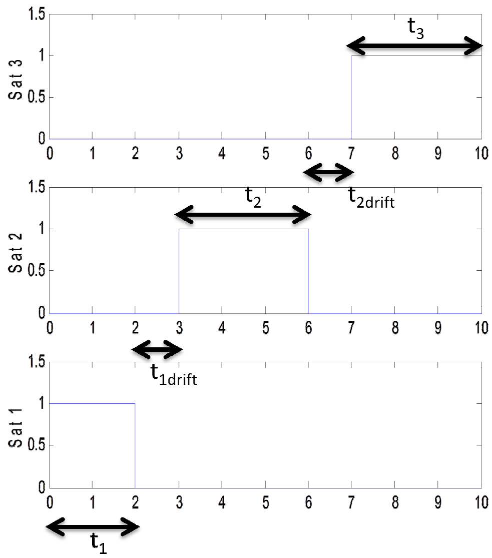
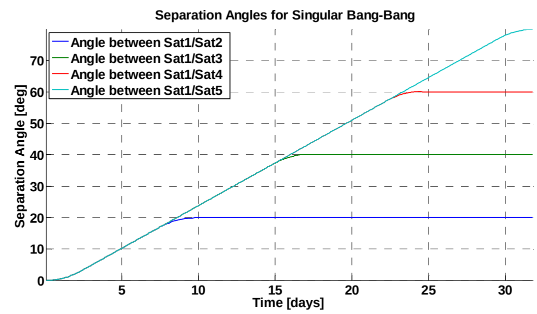


Figure 7 Concept of SBB control with 3 satellites

The SBB concept is fairly simple: since each satellite can only perform one maneuver, it would have to ideally inject itself into the correct orbit at the correct time. Combined with Taylor equations 6 and 7, and using the boundary conditions in equation 11, the problem simplifies to be dependent only upon one variable: we can either choose t_1, the time the initial satellite spends in full drag mode, or t_{1drift}, the drift time between satellites 1 and 2. Fixing one of these values allows the drift and maximum drag times (t_{idrift} and t_i) to be solved for all satellites. We then perform the similar utility transform *f (*we again use the HBT), and take the summation of all these values, analogous to integration, to determine total utility. We repeat the process for a range of t_i, and the solution is naturally concave; too much control leads to an immediate loss of orbit lifetime whereas too little control allows too much drift time, which in turn saps away the operational lifetime. The optimized SBB control solution is one in which, again, the total utility is maximized. The optimized SBB solution (using the HBT utility function) is shown in Figure 8.

As can be seen in Figure 8, the SBB is similar to the optimal control HBT solution, except for the control durations for maximum drag are evenly spread out. We see that instead of allowing the solution to drift towards the desired separation angle, we must force the separation angle to its nominal at a specific time. The solution is expectedly sub-optimal, and the total time necessary to achieve nominal takes a few more days. The SBB approach simply trades operational lifetime for reduced operational complexity.



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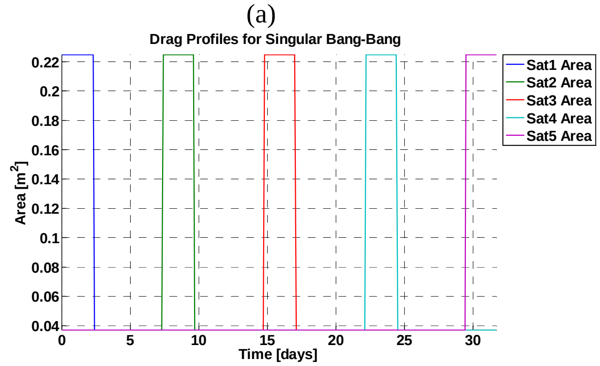


(b)

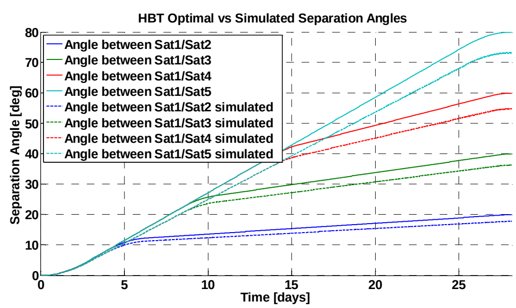
Figure 8 Optimal solution for SBB control (a) Separation Angle vs Time (b) Drag Profile vs Time

IV. Simulations

To evaluate these solutions in the presence of higher order perturbations, we simulate the orbits using Orekit. In addition to the constant density assumption (we let this condition hold to exclusively compare our current results with additional forces), we add spherical harmonics to the 10^{th} order as well as solar radiation pressure. The simulated results for the HBT optimal control case are shown in Figure 9a as the dashed lines.

We see from Figure 9a that the simulated solution increasingly diverges from the proposed theoretical solution over time. We expect some deviation since the circular assumption that we made previously does not hold as well under the addition of forces, as the orbits tend to be at least slightly elliptical. Additionally, the equations developed earlier considered only the average change in velocity, while the actual change in velocity possesses higher order harmonics.

However, we may be more interested in the difference in angles of successive satellites, as that is the basis for the utility function that we are integrating over. From Figure 9b we see that the separation angles converge to roughly 18°, which is 2° short of our target phasing. Recall that we utilized the HBT utility function, and that a separation angle close to nominal has approximately the same value as the nominal itself. In this respect the simulation has acceptable tolerances, especially for a numerical simulation over such a lengthy period of time.



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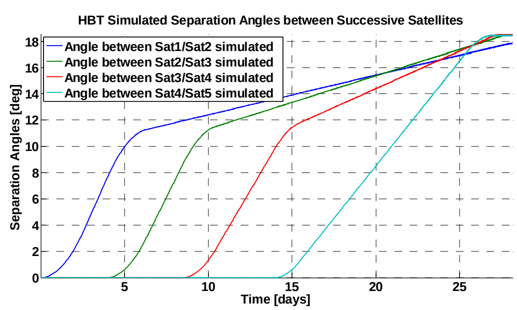
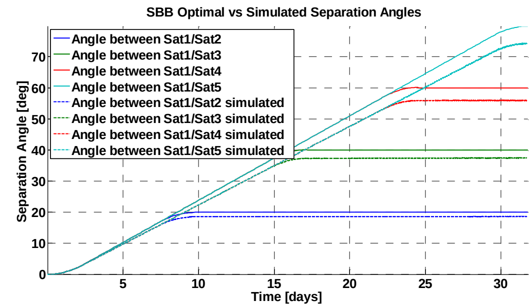


(b)

Figure 9 Optimal Solution vs Orekit Simulation (a) Separation angles between all satellites and 1st satellite (b) Separation angle between successive satellites

We will now also simulate under the same conditions the Taylor series SBB solution. The results are shown in Figure 10. Again we see the simulated solution falling short of the nominal angle of separation. Curiously, we see that the final separation angle between successive satellites approach the same value as that of the previous simulation.



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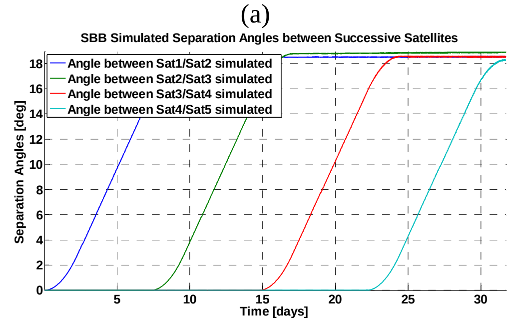

(b)

Figure 10 SBB Solution vs Orekit Simulation (a) Separation angles between all satellites and 1st satellite (b) Separation angle between successive satellites

Table 1 shows the comparison between the optimal control and SBB approaches, as evaluated (as per equation 12) using both the Taylor series formulation and numerical simulations. As expected, the optimal solution gives the highest value, followed by the theoretical Taylor SBB solution. We see that our simulated result is quite suboptimal due to the fact that we do not reach our final optimal values. Surprisingly the simulated bang-bang solution performs better than that of the simulated optimal solution, again due to perturbing forces.

Method	Total Utility [deg*days]	Relative performance
Taylor optimal/HBT	2649.1	100%
Numerical optimal/HBT	2159.6	81.5%
Taylor SBB/HBT	2574.3	97.2%
Numerical SBB/HBT	2247.2	84.8%

Table 1 Total utility (Eq. 9) comparison of control approaches and simulations methods.

To remedy the issue of the separation angle not approaching nominal in the simulated case, we can again use Taylor equations 6 and 7 close to the point where we are ready to make our final maneuver.

Another method is to use differential control near the final switching point, such that we ease into our desired orbit. The second method is preferable if our change in separation angle is large. Both these methods have been shown to work, albeit at the cost of decreasing our total utility.

An interesting side effect of this phasing technique is a difference of the ascending node between the orbits. This effect is inescapable, as we have no control over the cross-track motion. At the simulated altitudes, the cumulative effect of J2 at differing altitudes results in a longitude of ascending node angle difference of roughly 1° over 2 months.

V. Conclusions

We have developed here an optimal control formulation of the initial orbit phasing problem for a constellation of satellites, using only differential drag. We make the constant atmospheric density and circular orbit assumptions to derive our state equations. To evaluate control approaches a reasonable utility function must be posed that reflects the usefulness of a particular angle of separation between consecutive satellites. Three alternatives were compared, and it was found that the hyperbolic tangent (HBT) utility function works particularly well and captures the nature of such a utility evaluation. The optimal solution when maximizing total utility when using the HBT utility function is a bang-bang solution which is convenient to implement on satellites in orbit.

We also developed the Taylor approximation to the analytical equations, recognizing that the constant atmospheric density assumption is untrue for such a long period of time. As well, this formulation allows us to look at another set of solutions simpler than that of the optimal: the SBB solution where each satellite is only allowed 1 maximum drag maneuver throughout its acquisition phase. The formulation is successful due to the fact that the separation angle change very closely approximates a linear function over time. Due to the nature of the boundary conditions and equations, the solution is found by searching over an initial maximum drag duration that eventually yields all other times and durations of maximum drag of the remaining satellites. The formulation can be extrapolated to different atmospheric densities over discrete time periods, given that we are able to accurately model such changes. The solution is expectedly sub-optimal, but yields another practical solution to on-orbit drag maneuvers.

Simulation results show that the addition of perturbing forces such as spherical harmonics and solar radiation pressure lowers the expected optimal. However, the angle of separation between successive satellites do converge on a value close to optimal, and given our utility function, it is within reason. Remedies for the situation include deriving a Taylor series formulation close to the final switching point such that we switch at a later time, or introducing a differential control method close to this location.

Future work involves evaluating control methods against a realistic non-constant drag model, and the additional investigation of station keeping methods. As well, there exists inherent dependencies on many estimated parameters within our equations, such as atmospheric density or drag coefficients. Analyzing the sensitivity of the solution to changes in these unknowns is another area that needs to be addressed. Currently, the SBB method is preferred for the acquisition phase due to its ease of implementation. On orbit tests and validation will be performed once the Flock 1 constellation is in orbit, in parallel with the development of more advanced station-keeping control approaches.

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VI. Appendix

This paper is based upon differential drag of Cubesats, and as such the following values are used for simulation:

$m = 4.9\ kg$	(Mass)
$C_D = 2.2$	(Drag coefficient)
$\rho = 2.5 * 10^{-12}\ \frac{kg}{m^3}$	(Atmospheric density)
$A_{max} = 0.225\ m^2$	(Maximum area)
$A_{min} = 0.0371\ m^2$	(Minimum area)
$H_{initial} = 400\ km$	(Initial altitude)
$v_{initial} = 7668\ \frac{m}{s}$	(Initial velocity)
$\theta_{target} = \frac{\pi}{9}\ rad$	(Optimal separation angle)

Non-dimensionalization of the parameters is used to avoid numerical issues during the search for the optimal solution:

$\tau = \frac{t}{4*10^7}$ (Time)

$\omega = \frac{v}{1000}$ (Velocity - converts to km/s)

$\Theta = \frac{180}{\pi}\theta$ (Separation Angle - converts to deg)

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